New quantum aspects of a vacuum-dominated universe

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In a new model that we proposed, nonperturbative vacuum contributions to the effective action of a free quantized massive scalar field lead to a cosmological solution in which the scalar curvature becomes constant after a time t_j (when the redshift $z\sim 1$) that depends on the mass of the scalar field and its curvature coupling. This spatially flat solution implies an accelerating universe at the present time and gives a good one-parameter fit to high-redshift type Ia supernovae (SNe-Ia) data, and the present age and energy density of the universe. Here we show that the imaginary part of the nonperturbative curvature term that causes the cosmological acceleration implies a particle production rate that agrees with predictions of other methods and extends them to non-zero mass fields. The particle production rate is very small after the transition and is not expected to alter the nature of the cosmological solution. We also show that the equation of state of our model undergoes a transition at t_j from an equation of state dominated by non-relativistic pressureless matter (without a cosmological constant) to an effective equation of state of mixed radiation and cosmological constant, and we derive the equation of state of the vacuum. Finally, we explain why nonperturbative vacuum effects of this ultralow-mass particle do not significantly change standard early universe cosmology.

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I. INTRODUCTION

Ever since the path-breaking prediction by Casimir [1] that a quantized field in its vacuum state will exert a force on nearby conducting plates, and its confirmation in the laboratory [2], the fundamental importance of the vacuum has been clear. The electromagnetic vacuum also manifests itself through other observable effects such as the Lamb shift [3], the anomalous magnetic moment of the electron [4], and the anomaly-driven two-photon decay of the pion [5]. The predicted magnitudes of these effects are obtained through renormalization of fields and coupling constants, which absorb the infinities in a covariant manner. Renormalization methods have long been extended to quantized fields propagating in the curved spacetime of general relativity [6] (see also [7] and references therein), but predicted quantum vacuum effects of curved spacetime have been too small to be directly observed. Quantum vacuum effects in curved spacetimes have also been explored in connection with particle creation by the expansion of the universe and by black holes [8,9]. In addition, vacuum effects of self-interacting scalar fields have been invoked to produce inflation of the very early universe [10].

As is well known, observations of an accelerating expansion of the recent universe [11,12], together with other observations, such as those of small-angular-scale fluctuations of the cosmic microwave background radiation (CMBR), imply the existence of a dark form of energy [13–15]. The present authors have argued that the recent acceleration of the universe is the first directly observable manifestation of quantum vacuum effects produced by the curved spacetime of general relativistic cosmology, and have presented a cosmological model, based on general relativity and quantum field theory, that fits the current data [16–18]. The position of the first acoustic peak of the CMBR fluctuation spectrum also agrees with observation, as explained in the last paragraph of the Conclusions section. These quantum vacuum effects involve an ultralow-mass particle and are nonperturbative in the curvature. They do not become significant until a transition that occurs at about half the present age of the universe (i.e., at a redshift $z\sim 1$).

In the present paper, we explore new features of our model, and their possible cosmological significance. In particular, we calculate the creation rate of these ultralight particles resulting from the effect of the spacetime curvature on the vacuum. The analytic result for the particle production rate during the vacuum-dominated stage predicted by our model reduces to that obtained in the massless limit by other methods, and extends the result to the massive case. This agreement adds confidence to the validity of our effective action and its other predictions, such as that of a recent acceleration of the expansion of the universe. *The predicted acceleration and the particle production arise from the real and imaginary parts, respectively, of the same term in the effective action.*

We then find the effective temperature of the particles created during the vacuum-dominated stage. This effective temperature at the present time would be very small (only about 10^{-112} K), and is not expected to alter the vacuum-dominated behavior. It should be kept in mind that the remnant of these particles left over from the very early universe would have a much higher temperature today, probably of the order of 1 K.

The mass of the ultralight scalar particle in our model is typical of masses expected for pseudo Nambu-Goldstone bosons (PNGB). In Ref. [15] the PNGB mass scale arises from the ratio of m_ν^2 to f ($m_\nu\sim 10^{-3}$ eV is a neutrino mass and $f\sim 10^{18}$ GeV is a global symmetry-breaking scale), quantities that are *a priori* independent of the present expansion rate of the universe. If the mass comes from a PNGB

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mechanism, then this may introduce additional interaction terms into our free-field model. Also, it is known that in a Friedmann-Robertson-Walker (FRW) universe, the graviton field equation can be expressed in the form of two scalar field equations. If the graviton has an ultralow mass, then we expect that it would lead to the same cosmological consequences as the scalar particle discussed here. Furthermore, such a low graviton mass appears to be consistent with gravitational experiments and observations.

The expansion rate being related to the ultralow mass is not a coincidence in our model, because after the transition, such a relation between the mass of the particle and the expansion rate remains true for a time of the order of the present age of the universe. In addition, the vacuum energy density remains within an order of magnitude of the matter density for a time of the order of the present age of the universe.

The organization of this paper is as follows. In Sec. II, we briefly summarize our model and the cosmological solution that arises from it. In Sec. III, we discuss the particle production rate found from the imaginary part of the effective action. In Sec. IV, we discuss the fine-tuning problem in our model and in a model with cosmological constant plus nonrelativistic matter (referred to here as the Λ model). In Sec. V, we derive the equation of state of the vacuum in our model. In Sec. VI, we calculate the ratio of total (i.e., matter plus vacuum contributions) pressure and total energy density for our model and compare it to that for a Λ model. This ratio as function of redshift is useful for future comparison to observations. In Sec. VII, we discuss nonperturbative quantum vacuum effects of this ultralow-mass field in the early universe, and show that the standard cosmological model is not significantly altered for times less than about half the present age of the universe. Finally, in Sec. VIII, we state our conclusions.

II. NONPERTURBATIVE VACUUM ENERGY EFFECTS

We start with a brief summary of our non-perturbative vacuum energy model and our previous results [16–18]. Consider a free, massive quantized scalar field of inverse Compton wavelength m, and curvature coupling ξ. The effective action for gravity coupled to such a field is obtained by integrating out the vacuum fluctuations of the field [16–20] and renormalized as in Ref. [16]. This effective action is the simplest one that gives the standard trace anomaly in the massless-conformally-coupled limit, and contains the nonperturbative sum (in arbitrary dimensions) of *all* terms in the propagator having at least one factor of the scalar curvature, R. It is given by [16]

$$W=\int d^4x\sqrt{-g}\,\kappa_o R+\frac{\hbar}{64\pi^2}\int d^4x\sqrt{-g}\left[-m^4\ln\left|\frac{M^2}{m^2}\right|+m^2\bar{\xi}R\left(1-2\ln\left|\frac{M^2}{m^2}\right|\right)-2f_2\ln\left|\frac{M^2}{m^2}\right|+\frac{3}{2}\bar{\xi}^2R^2\right]+\frac{i\hbar}{64\pi}\int d^4x\sqrt{-g}(M^4+2\bar{f}_2)\,\theta(-M^2), \tag{1}$$

where $\kappa_o\equiv(16\pi G)^{-1}$ (G is Newton's constant), $\bar{\xi}\equiv\xi-1/6$, and

$$M^2\equiv m^2+\bar{\xi}R \tag{2}$$

$$\bar{f}_2\equiv(1/6)(1/5-\xi)\Box R+(1/180)\times(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}-R_{\alpha\beta}R^{\alpha\beta}) \tag{3}$$

$$f_2\equiv\bar{f}_2+(1/2)\bar{\xi}^2R^2. \tag{4}$$

The renormalized cosmological constant has been set to zero in deriving the above effective action. The last term in the above equation constitutes the imaginary part of the effective action, implying particle production. We will discuss the magnitude of the imaginary part in the next section. For now, we focus on the real part of the effective action.

The trace of the Einstein equations, obtained by variation of the real part of the effective action with respect to the metric tensor takes the following form in a Friedmann-Robertson-Walker (FRW) spacetime (in units such that $c=1$):

$$R+\frac{T_{cl}}{2\kappa_o}=\frac{\hbar m^2}{32\pi^2\kappa_o}\left\{(m^2+\bar{\xi}R)\ln|1+\bar{\xi}Rm^{-2}|-\frac{m^2\bar{\xi}R}{m^2+\bar{\xi}R}\times\left(1+\frac{3}{2}\bar{\xi}\frac{R}{m^2}+\frac{1}{2}\bar{\xi}^2\frac{R^2}{m^4}(\bar{\xi}^2-(1080)^{-1})+v\right)\right\}, \tag{5}$$

where T_{cl} is the trace of the energy-momentum tensor of classical, perfect fluid matter and $v\equiv(R^2/4-R_{\mu\nu}R^{\mu\nu})/(180m^4)$ is a curvature invariant that vanishes in de Sitter space. As a result of including the nonperturbative sum of scalar curvature terms in the effective action, the right hand side of Eq. (5) becomes large as $R\rightarrow m^2/(-\bar{\xi})$. This resonance can occur even at low curvatures if m^2 is sufficiently small, and is not displayed by a perturbative treatment of R; as can be seen by expanding Eq. (5) in powers of R/m^2 and keeping a finite number of terms.

As noted earlier, m is the inverse Compton wavelength of the field. It is related to the actual mass of the field by $m_{\rm actual}=\hbar m$. Equation (5) above is nonperturbative in R because it contains terms that involve an infinite sum of powers of R. However, for a sufficiently low mass, it is possible to treat $m_{\rm actual}^2/m_{Pl}^2\equiv\hbar m^2/(16\pi\kappa_o)$ (where m_{Pl} is the Planck mass) as a small parameter and expand perturbatively in this parameter.

For a sufficiently low mass, in an expanding FRW universe the quantum contributions to the Einstein equations become significant at a time t_j, when the density of classical matter, ρ_m, has decreased to a value given by

$$\rho_m(t_j)=2\kappa_0\overline{m^2}, \tag{6}$$

where

$$\overline{m^2}\equiv m^2/(-\bar{\xi}). \tag{7}$$

The time t_j occurs in the matter-dominated stage of the evolution. Furthermore for $t>t_j$, the scalar curvature, R, remains constant to excellent approximation near the value $\overline{m}^2$. For $t<t_j$, the quantum contributions to the Einstein equations are negligible and the scale factor is that of a matter-dominated FRW universe. Then, Eq. (6) implies that, in a spatially flat universe with line element $ds^2=-dt^2+a(t)^2(dx^2+dy^2+dz^2)$, one has

$$t_j=(2/\sqrt{3})\overline{m}^{-1}, \quad H(t_j)=\overline{m}/\sqrt{3}, \tag{8}$$

where $H(t_j)$ is the Hubble constant at t_j.

The condition of the constancy of the scalar curvature after t_j leads to a solution for the scale factor that can be joined, with continuous first and second derivatives (i.e., in a C^2 manner), to the matter-dominated solution for $t<t_j$. The scale factor is given by

$$a(t)=a(t_j)\sqrt{\sinh\left(\frac{t\overline{m}}{\sqrt{3}}-\alpha\right)\Big/\sinh\left(\frac{2}{3}-\alpha\right)}, \quad t>t_j,$$

$$=a(t_j)(\sqrt{3}\,\overline{m}t/2)^{2/3}, \quad t<t_j, \tag{9}$$

with

$$\alpha=2/3-\tanh^{-1}(1/2)\simeq 0.117. \tag{10}$$

We would like to point out here that the above solution also satisfies, up to terms of order $m_{\rm actual}/m_{Pl}$, the one remaining independent Einstein equation in a FRW universe, which can be taken to be $G_{00}=(2\kappa_o)^{-1}T_{00}$, where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the energy-momentum tensor, including classical and quantum contributions. This equation takes the form, with zero cosmological constant,

$$k_oG_{00}=\frac{1}{2}\rho_m-\frac{\hbar}{64\pi^2}\left\{\frac{\overline{\xi}R_{00}}{m^2+\overline{\xi}R}\left(m^4+2m^2\overline{\xi}R+\frac{R_{\alpha\beta}R^{\alpha\beta}}{90}+R^2\left(\overline{\xi}^2-\frac{1}{270}\right)\right)-3\overline{\xi}^2R\,R_{00}+m^2\overline{\xi}G_{00}\right\}$$
$$-\frac{\hbar}{64\pi^2}\ln\left|1+\frac{\overline{\xi}R}{m^2}\right|\left\{-\frac{m^4g_{00}}{2}+2m^2\overline{\xi}G_{00}-\frac{g_{00}R^2}{2}\left(\overline{\xi}^2+\frac{1}{90}\right)+\frac{1}{90}g_{00}R_{\alpha\beta}R^{\alpha\beta}+2\overline{\xi}^2R\,R_{00}-\frac{2}{45}R_0^{\alpha}R_{0\alpha}\right\}. \tag{11}$$

To verify that Eq. (9) is indeed a solution of the above equation for $t>t_j$, we note that, when R is very close to the value $\overline{m}^2$, the dominant terms in the right-hand side of Eq. (11) are those that have a factor of $m^2+\overline{\xi}R$ in the denominator. Keeping these terms and substituting for the various curvature quantities derived from Eq. (9), one finds that Eq. (9) satisfies Eq. (11) up to terms of order $m_{\rm actual}/m_{Pl}$.

The solution in Eq. (9) corresponds to a universe that is accelerating (i.e., has $\ddot{a}>0$) for $t>\sqrt{3}\overline{m}^{-1}[\alpha+\tanh^{-1}(2^{-1/2})]\simeq 1.50\,t_j$. This solution gives a good fit [11,12] to the SNe-Ia data, for the mass range

$$6.40\times10^{-33}\,\text{eV}<\left(\frac{\overline{m}}{h}\right)<7.25\times10^{-33}\,\text{eV}, \tag{12}$$

where h is the present value of the Hubble constant, measured as a dimensionless fraction of the value 100 km/(s Mpc).

The ratio of matter density to critical density at the present time, Ω_0, is a function of the single parameter $\overline{m}/h$ and turns out to have the range $0.58>\Omega_0>0.15$ for the range of values of Eq. (12). For the same range of values, the age of the universe t_0 lies in the range $8.10\,h^{-1}$ Gyr $<t_0<12.2\,h^{-1}$ Gyr. These ranges for Ω_0 and t_0 agree with current observations [21].

III. PARTICLE PRODUCTION AND EFFECTIVE TEMPERATURE

In this section, we consider the rate of particle production in the cosmological solution discussed above. We find that the effective action from which our cosmological solution is derived leads to a particle production rate that is consistent with that obtained using other methods, and generalizes the other methods to the case of production of massive particles. We also discuss the effective temperature of these particles.

The constant-R solution after the transition at time t_j was obtained from a consideration of the Einstein equations based on variation of the real part of the effective action. The imaginary part of the effective action is related to the probability of the production of at least one pair of particles [8], following Schwinger [22]. When the imaginary part is small, this probability is given by

$$P=2\frac{\text{Im}\,W}{\hbar}. \tag{13}$$

From Eq. (1), we obtain

$$\frac{\text{Im}\,W}{\hbar}=(64\pi)^{-1}\int d^4x\sqrt{-g}(M^4+2\overline{f}_2)\,\theta(-M^2). \tag{14}$$

When $M^2<0$ and $m=0$, the above formula reduces to that derived in Refs. [23] and [24]. The derivation of the particle production rate in [24] is based on the non-local effective action derived in [25], which contains, in the $m\to0$ limit, terms of the form $\ln(\Box)$. Similar terms were discussed in [26]. It is noteworthy that the nonperturbative scalar curvature sum used here gives the same imaginary part, in the limiting case of zero mass, as the nonlocal effective action.

The reason for the similar results in the two cases was suggested in [26], based on renormalization group arguments [27]. However, these renormalization group arguments are generally reliable only in the large-R limit. The partially summed form of the effective action that we use here is also valid at small values of R, as in the present universe.

For $m \neq 0$, Eq. (14) agrees, to leading order, with the particle production rate derived in [28]. Since the imaginary and real parts of the effective action arise out of the same term, the agreement of the imaginary part with that found in the literature using other methods gives a further justification of our approximation.

For our cosmological solution, $M^2 < 0$ for all time, therefore the step function in the above equation is equal to 1. The term $\overline{f_2} \equiv (1/6)(1/5-\xi)\Box R + (1/180)(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - R_{\alpha\beta}R^{\alpha\beta})$ can be expressed as

$$\overline{f_2} = \frac{1}{6}\left(\frac{1}{5}-\xi\right)\Box R + \frac{1}{120}C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta} - \frac{1}{360}G, \quad (15)$$

where $C_{\alpha\beta\gamma\delta}$ is the Weyl tensor, and G is the Gauss-Bonnet invariant

$$G = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2. \quad (16)$$

In conformally flat spacetimes, such as the one under consideration, the Weyl tensor vanishes. Therefore $\overline{f_2}$ is a total derivative term,[1] and $\int d^4x\sqrt{-g}\overline{f_2}$ only has boundary contributions. Since these boundary contributions would vanish if the metric were static at the boundaries, one expects that the $\overline{f_2}$ term in the imaginary part of the effective action does not correspond to real particle production [29]. The remaining term, M^4, will be used to estimate the probability of particle production.

From Refs. [16] and [17], we find, for $t > t_j$,

$$M^4 \simeq (4320\pi)^{-2}\left(\frac{m}{m_{\rm Pl}}\right)^4 \overline{m}^4. \quad (17)$$

From Eqs. (14) and (17), we obtain the probability per unit proper volume, p, for production of at least one pair of particles after time t_j, as

$$p \simeq (32\pi)^{-1}(4320\pi)^{-2}\left(\frac{m}{m_{\rm Pl}}\right)^4 \overline{m}^4 a(t)^{-3}\int_{t_j}^{t} dt'\, a(t')^3. \quad (18)$$

To find the effective temperature of the particles produced, we compare the above expression with the case when the particles are produced with a thermal spectrum.[2] For thermal production, we have [24]

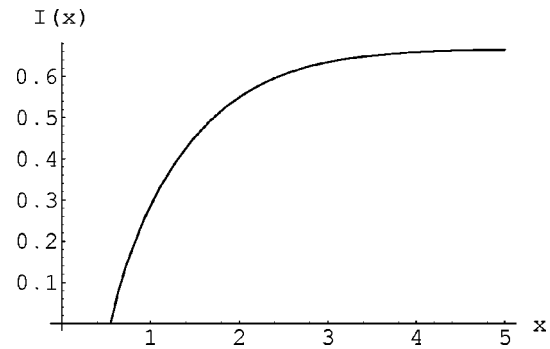

FIG. 1. A plot of $\mathcal{I}(x)$ versus $x = \overline{m}ct/3^{1/2} - \alpha$. The graph begins at $t = t_j$ and asymptotes to a value of 2/3 as $x \to \infty$. The present time corresponds to $\mathcal{I}(x) \simeq 0.5$.

$$p = \frac{\pi^2}{90}(k_B T)^3, \quad (19)$$

where T is the physical, or measured, temperature, and k_B is Boltzmann's constant. From Eqs. (18) and (19), we obtain the effective physical temperature of the particles produced, as

$$k_B T = \hbar \overline{m} c^2 \left(\frac{90\sqrt{3}}{32(4320)^2\pi^5}\right)^{1/3}\left(\frac{m}{m_{\rm Pl}}\right)^{4/3}\mathcal{I}(\overline{m}ct)^{1/3}, \quad (20)$$

where we have now inserted appropriate factors of c, and $\mathcal{I}(\overline{m}ct)$ is the dimensionless function

$$\mathcal{I}(\overline{m}ct) = (\sinh x)^{-3/2}\int_{2/3-\alpha}^{x} dy\,(\sinh y)^{3/2}, \quad (21)$$

with $x = \overline{m}ct/\sqrt{3} - \alpha$. Note that the factor of $\hbar$ that appears in Eq. (20) above is necessary because $\overline{m}$ is an inverse length scale, rather than a mass. A plot of $\mathcal{I}(x)$ vs x is shown in Fig. 1. It is easy to verify analytically that $\mathcal{I}(\infty) = 2/3$. Also, at the present time, t_0 [defined as the time at which the Hubble constant has the value 65 km/(s Mpc)], one obtains $\mathcal{I}(\overline{m}ct_0/\sqrt{3} - \alpha) \simeq 0.5$.

As shown earlier, a good fit to the SNe-Ia data is obtained when the mass parameter m_h has the value 6.93×10^{-33} eV. With the rescaled Hubble parameter, $h = 0.65$, this gives a value $\overline{m} = 4.5045 \times 10^{-33}$ eV. Also, $m_{Pl} \simeq 1.2211 \times 10^{28}$ eV. One can reexpress the effective temperature as

$$T(\overline{m}ct) \simeq 1.31 \times 10^{-112}\,\mathrm{K}\;\mathcal{I}(\overline{m}ct)^{1/3}\left(\frac{\overline{m}}{4.5045 \times 10^{-33}\ \mathrm{eV}}\right) \times \left(\frac{m}{4.5045 \times 10^{-33}\ \mathrm{eV}}\right)^{4/3}. \quad (22)$$

If m and $\overline{m}$ are roughly equal to 4.5045×10^{-33} eV, then the above temperature corresponds to the temperature of the Hawking radiation emitted by a Schwarzschild black hole of mass $\simeq 10^{138}$ g, which is about 10^{105} solar masses.

[1] $\Box R$ is obviously a total derivative, and in Robertson-Walker spacetimes, $\sqrt{-g}G = 24(d/dt)(\dot{a}^3/3 + k\dot{a})$, which is a total time derivative ($k = 0, \pm 1$, corresponding to flat, closed or open spatial sections).

[2] Although the actual spectrum may not be thermal, the high-frequency behavior of the spectrum is expected to be so [30–32].

The fact that the one can associate a temperature with the created particles does not mean that the vacuum possesses thermodynamic entropy. Indeed, from the equation of state of the vacuum, it is straightforward to check that the change of entropy in a comoving volume of space a^3V is given by $TdS=d(\rho_V a^3V)+p_V d(a^3V)$, which vanishes as a consequence of conservation of vacuum energy.

IV. THE FINE-TUNING PROBLEM

In the absence of data that does not distinguish between models for an accelerating universe, a criterion for the success of a given model is the lack of fine-tuning of fundamental parameters. In a spatially flat model with cosmological constant, Λ, and non-relativistic matter (Λ model), the fine-tuning problem is often understood as a coincidence between the matter density ρ_m and the energy density associated with the cosmological constant, ρ_Λ, at the present time. However, it is straightforward to show, in both our model and the Λ-model, that the time interval for which ρ_m is, say, between 0.1 and 0.9 times the critical density, ρ_c is of the order of the age of the universe. This is also the time interval for which ρ_m and the vacuum energy density[3] ρ_V are roughly within an order of magnitude of each other. The calculation runs as follows. In our model, Ω_0 and ht_0 are the following functions of the single parameter $m_h \equiv \overline{m}/h$ [17]:

$$\Omega_0=(2.996\times 10^6\ \ \mathrm{Mpc}^2)m_h^2\left(\frac{\sinh(cm_h ht_0/\sqrt{3}-\alpha)}{\sinh(2/3-\alpha)}\right)^{-3/2} \tag{23}$$

$$ht_0=3.26\times 10^6\frac{\mathrm{yr}}{\mathrm{Mpc}}\left(\frac{\sqrt{3}}{m_h}\right)[\tanh^{-1}(865.4\ \ \mathrm{Mpc}\, m_h)+\alpha], \tag{24}$$

where m_h is in units of Mpc^{-1}. If $0.1<\Omega_0<0.9$, then the above equations give $1.143\times 10^{-3}\,\mathrm{Mpc}^{-1}>m_h>7.955\times 10^{-4}\,\mathrm{Mpc}^{-1}$. For this range of m_h values, one then obtains $13.4\,\mathrm{Gyr}>ht_0>6.83\,\mathrm{Gyr}$. Thus the time interval for which $0.1<\Omega_0<0.9$ is $(13.4-6.83)h^{-1}$ Gyr, or $6.57\,h^{-1}$ Gyr, which is of the order of the age of the universe.

In the Λ model, it is straightforward to show that Ω_0 and ht_0 are functions of the single parameter $C\equiv\sqrt{6\pi G\rho_\Lambda}/(ch)$, which has dimensions of $(\mathrm{time})^{-1}$ (with the rescaled Hubble parameter h regarded as a dimensionless quantity). They are

$$\Omega_0=1-\frac{4}{9}(C\,9.78\ \ \mathrm{Gyr})^2 \tag{25}$$

$$ht_0=C^{-1}\tanh^{-1}\left(\frac{2}{3}C\,9.78\ \ \mathrm{Gyr}\right). \tag{26}$$

One similarly finds, for $0.1<\Omega_0<0.9$, that $12.49\,\mathrm{Gyr}>ht_0>6.75\,\mathrm{Gyr}$, giving a time interval of $5.74h^{-1}$ Gyr, again of the order of the age of the universe.

The preceding arguments show that there is no ''coincidence'' problem in the sense of the present time being special in either model. However, in the case of the Λ model, an alternative statement of the fine-tuning problem is that there is no ''natural'' explanation, within elementary particle physics, of a small non-zero value of ρ_Λ. The favored values of ρ_Λ are either 0 or m_{Pl}^4, the latter value being at discrepancy with observations by about 122 orders of magnitude.

In our model, the question then is whether there may be a natural explanation for the small value of the mass parameter $\overline{m}\simeq H_0$. Frieman *et al.* [15] show that, for spin-0 pseudo Nambu-Goldstone bosons (PNGB), a mass scale $\overline{m}$ of the order of the present value of the Hubble constant can be generated from the neutrino mass $m_\nu\simeq 10^{-3}$ eV and a global symmetry breaking scale [for example, the scale for spontaneous breaking of the U(1) Peccei-Quinn symmetry] $f\simeq 10^{18}$ GeV, via the combination $\overline{m}\simeq m_\nu^2/f$. They also show that this combination arises out of the coupling of such a pseudo Nambu-Goldstone boson to neutrinos in a low-energy effective theory. It therefore appears possible that the ultralight spin-0 particle we consider in our model may be related to a PNGB, although such a relation would involve additional self-interaction terms.

V. VACUUM EQUATION OF STATE

Although the basic dynamical equations, Eqs. (2) and (8), incorporate non-trivial quantum effects, our model admits a remarkably simple description. Indeed, the scale factor (9) may be used to find the total effective energy density ρ and pressure p of vacuum plus matter by directly computing the Einstein tensor. We obtain, for $t>t_j$,

$$\begin{aligned}\rho(t)&=2\kappa_0 G_{00}=(\kappa_0\overline{m}^2/2)\coth^2(t\overline{m}/\sqrt{3}-\alpha)\\&=(3/2)\kappa_o\overline{m}^2[a(t)/a(t_j)]^{-4}+(1/2)\kappa_o\overline{m}^2\end{aligned} \tag{27}$$

$$\begin{aligned}p(t)&=2\kappa_0 a(t)^{-2}G_{ii}\\&=(\kappa_0\overline{m}^2/6)\coth^2(t\overline{m}/\sqrt{3}-\alpha)-(2/3)\kappa_0\overline{m}^2.\end{aligned} \tag{28}$$

The effective equation of state for $t>t_j$ is therefore

$$p=(1/3)\rho-(2/3)\kappa_0\overline{m}^2, \tag{29}$$

which is identical to the equation of state for a classical model consisting of radiation plus cosmological constant. In our model, the equation of state of pressureless matter and the equation of state of quantum vacuum terms combine so as to appear as a sum of radiation and cosmological constant equations of state. Our model differs, even at the classical level, from the usual mixed matter-cosmological constant model because (i) for $t<t_j$ the effective cosmological constant vanishes, and (ii) for $t>t_j$ vacuum contributions transmute the effective equation of state into that of *radiation* (rather than pressureless matter) plus cosmological constant;

[3]We will often refer to the vacuum energy density as ρ_V, independent of the model under consideration. For the Λ model, $\rho_V=\rho_\Lambda$.

this surprising metamorphosis is a result of the near-constancy of the scalar curvature, which causes certain terms in $T_{\mu\nu}$ to take the form of an effective cosmological constant term in Einstein's equations. In a *general* spacetime, these terms do *not* have the form of a cosmological constant term.

The equation of state for the quantum vacuum terms *alone* may be inferred from Eqs. (27) and (18), and from the fact that the density of pressureless matter is given by

$$\rho_m(t)=\rho_m(t_j)\,[a(t_j)/a(t)]^3 = 2\kappa_0\overline{m}^2[\sinh(2/3-\alpha)/\sinh(t\overline{m}/\sqrt{3}-\alpha)]^{3/2}, \tag{30}$$

where Eqs. (6) and (9) have been used to arrive at the second equality. The quantum vacuum energy density ρ_V and pressure p_V then follow, for $t>t_j$, as

$$\rho_V(t)=\rho(t)-\rho_m(t) = \frac{\kappa_0\overline{m}^2}{2}\left[\coth^2(t\overline{m}/\sqrt{3}-\alpha) - 4\left(\frac{\sinh(2/3-\alpha)}{\sinh(t\overline{m}/\sqrt{3}-\alpha)}\right)^{3/2}\right] \tag{31}$$

$$p_V(t)=p(t), \tag{32}$$

with $p(t)$ given by Eq. (18). The above equations show that for $t>t_j$ the vacuum energy density is positive, while the vacuum pressure is negative. As stated earlier, the vacuum terms are negligible for $t<t_j$.

As a consequence of Eqs. (13) and (14), we find that the equation of state for the vacuum is

$$\rho_V=3p_V+2\overline{m}^2\kappa_0[1-(1+2p_V/(\kappa_0\overline{m}^2))^{3/4}]. \tag{33}$$

This vacuum equation of state joins continuously to the equation of state $\rho_V=p_V=0$ at $t=t_j$ and is asymptotic to the pure cosmological constant equation of state $\rho_V=-p_V$ as $t\to\infty$. Equation (33) is parametrized by the single parameter, $\overline{m}$, and is different from the equation of state of a pure cosmological constant. However, Eq. (33) holds only in the case of Robertson-Walker symmetry. Any inhomogenous perturbation of the matter content or the metric in our model would change the effective equation of state of the vacuum, since such perturbations would change the form of the right-hand-sides of Eqs. (2) and (8). This feature is a complication when one analyzes the evolution of small perturbations in the theory. However, we expect that the terms of Eq. (33) may still dominate the equation of state for the perturbations.

VI. RATIO OF PRESSURE AND ENERGY DENSITY

In this section, we derive the ratio of total pressure p and total matter density ρ in our model, as a function of redshift. This quantity, which we call $w(z)\equiv p/\rho$, is a useful one for comparison with future observations. We also compare $w(z)$ in our model with the same quantity in a model with non-relativistic matter plus cosmological constant.

From Eqs. (24) and (25) above, we find that, for $z<z_j$,

$$w(z)=\frac{1}{3}-\frac{2}{3}\frac{\kappa_o\overline{m}^2}{\rho}, \tag{34}$$

with

$$\rho(z)=\kappa_o\overline{m}^2\left\{\frac{3}{2}\left(\frac{1+z}{1+z_j}\right)^4+\frac{1}{2}\right\}. \tag{35}$$

The redshift at which the transition occurs, z_j, is given by

$$1+z_j\equiv\frac{a(t_0)}{a(t_j)}=\sqrt{\frac{\sinh(ct_0\overline{m}/\sqrt{3}-\alpha)}{\sinh(2/3-\alpha)}}, \tag{36}$$

where we have inserted an appropriate factor of c. Using the value of α from Eq. (7), and the present value of the Hubble constant,

$$H_0=\frac{c\overline{m}}{\sqrt{12}}\coth\left(\frac{ct_0\overline{m}}{\sqrt{3}}-\alpha\right), \tag{37}$$

one can reexpress Eq. (36) as

$$1+z_j=\left(\frac{4H_0^2}{c^2\overline{m}^2}-\frac{1}{3}\right)^{-1/4} =\left\{0.3788\left(\frac{6.93\times10^{-33}\ \text{eV}}{m_h}\right)^2-\frac{1}{3}\right\}^{-1/4}, \tag{38}$$

where we have substituted for the numerical value of the speed of light, c, in the second equality.

Combining Eqs. (34), (35) and (38), we obtain

$$w(z)=\frac{1}{3}-\frac{4}{3}\left\{1.1364(1+z)^4\left(\frac{6.93\times10^{-33}\ \text{eV}}{m_h}\right)^2-(1+z)^4+1\right\}^{-1}, \tag{39}$$

for $z<z_j$. For $z>z_j$, our model is entirely matter-dominated, therefore $p=w(z)=0$.

On the other hand, in a spatially flat model with non-relativistic matter plus cosmological constant, it is straightforward to show that

$$w(z)=-\{1+(1+z)^3(\Omega_\Lambda^{-1}-1)\}^{-1}, \tag{40}$$

where Ω_Λ is the ratio of the energy density in the cosmological constant and the critical density.

A plot of $w(z)$ for both models discussed above is shown in Fig. 2, with representative values of $m_h=6.93\times10^{-33}$ eV (in our model), and $\Omega_\Lambda=0.7$ (in the non-relativistic matter plus cosmological constant model). Future experimental data should be able to distinguish between these models.

Noting that the equation of state of non-relativistic matter would give $w(z)=0$, and the equation of state of a pure cosmological constant would give $w(z)=-1$, we see from

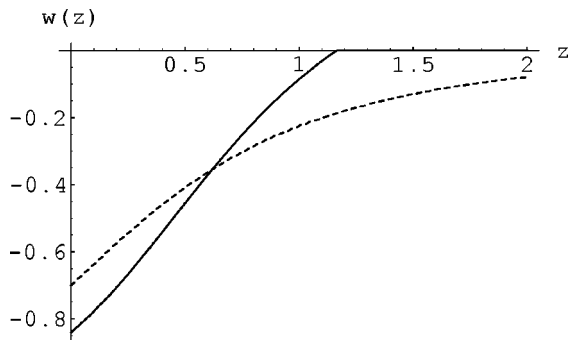

FIG. 2. A plot of $w(z)$ versus redshift z, in our model (solid curve) and a mixed matter plus cosmological constant model (dashed curve). The relevant parameter values are $m_h = 6.93 \times 10^{-33}$ eV (solid curve), and $\Omega_\Lambda = 0.7$ (dashed curve).

Fig. 2 that the redshift interval for which the effects of non-relativistic matter and vacuum energy are both significant is $\Delta z \simeq 2$. This range is a small one compared to the full redshift range, which is infinite. This feature is often taken to comprise an alternative statement of the ''coincidence'' problem outlined in the previous section. However, as we showed there, the small redshift range corresponds to a large range in time. The discrepancy between posing the ''coincidence'' problem in terms of time range and redshift range reflects the subjective nature of this problem. We reiterate that the most reasonable way to pose the fine-tuning problem is in terms of a fundamental explanation for the smallness of Λ or m_h.

VII. NONPERTURBATIVE EFFECTS OF AN ULTRALOW MASS FIELD IN THE EARLY UNIVERSE

In this section, we consider the possible quantum vacuum effects of an ultralow mass field ($m \sim 10^{-33}$ eV) in the early, post-inflationary universe. We first note that quantum vacuum effects become significant whenever the ''resonance'' condition is satisfied, i.e., whenever the trace of the classical stress tensor $T_{\rm cl} \equiv -\rho_{\rm cl} + 3p_{\rm cl}$ decreases below the value $2\kappa_o \overline{m^2}$ [see the arguments leading from Eq. (5) to Eq. (6) for details]. Consider now the evolution of $T_{\rm cl}$ from early inflation to the present time. During early inflation, $T_{\rm cl}$ has a large negative value, i.e., $|T_{\rm cl}| \gg 2\kappa_o \overline{m^2}$. After the reheating process at the end of inflation, the universe is expected to be nearly radiation-dominated, with $|T_{\rm cl}|$ decreasing to a small value. As the universe expands, and enough matter becomes non-relativistic, $|T_{\rm cl}|$ increases smoothly to a maximum value much greater than $2\kappa_o \overline{m^2}$. As the universe expands further, it enters a matter-dominated stage. During this stage, $|T_{\rm cl}|$ decreases again, until about half the age of the universe ($z \sim 1$), when it passes through the value $2\kappa_o \overline{m^2}$, and quantum vacuum effects begin to dominate, leading to the present acceleration. However, as described above, there are two early stages of the evolution during which $|T_{\rm cl}|$ may pass through the value $2\kappa_o \overline{m^2}$, namely, during the reheating from inflationary expansion to radiation-dominated expansion, and when $|T_{\rm cl}|$ is increasing during the radiation-dominated stage of the expansion of the universe. What role do quantum vacuum effects of this ultralow-mass particle play during these two early stages?

To answer this question, we consider two possibilities for the value of $|T_{\rm cl}|$ during the radiation-dominated stage: (a) there is sufficient non-relativistic matter at the end of post-inflationary reheating so that $|T_{\rm cl}|$ is always greater than $2\kappa_o \overline{m^2}$ during the radiation-dominated stage, or (b) there is not enough non-relativistic matter, and the condition, $|T_{\rm cl}| < 2\kappa_o \overline{m^2}$, is satisfied at some time during the radiation-dominated stage.

If possibility (a) occurs, then the ''resonance'' condition is never satisfied during the reheating from inflationary to radiation-dominated expansion, and during the subsequent evolution from radiation-dominated to matter-dominated expansion. Thus these quantum vacuum effects of an ultralow-mass particle remain negligible in the early universe, and during the matter-dominated stage, until the universe transits to vacuum-dominance, at about half its present age, as described in Sec. II.

If possibility (b) occurs, then a resonance of the type discussed here will occur during the reheating from inflationary expansion to radiation-dominated expansion, and will force R to remain nearly equal to $\overline{m^2}$ during the radiation-dominated stage. During such a stage, the scale factor would be $a(t) \propto \sqrt{\sinh(t\overline{m}/\sqrt{3})}$. However, $t\overline{m} \ll 1$ during the radiation-dominated stage, and the scale factor, to excellent approximation, exhibits radiation-dominated behavior, i.e., $a(t) \propto t^{1/2}$. Once enough non-relativistic matter has accumulated, $|T_{\rm cl}|$ increases above the value $2\kappa_o \overline{m^2}$ while the universe is still radiation-dominated. At this point, quantum effects become negligible, as can be seen from Eq. (5), and the universe eventually transits to the matter-dominated stage.[4] Much later, at time t_j, when the condition $|T_{\rm cl}| = 2\kappa_o \overline{m^2}$ is again satisfied, the universe transits to vacuum-dominance, giving rise to the present acceleration. However, because the density of non-relativistic matter (or equivalently, $|T_{\rm cl}|$) is now decreasing, the universe remains vacuum-dominated for all time.

The above arguments show that, even if these ultralow-mass-nonperturbative quantum vacuum effects become significant at early times in the evolution of the universe, the transition to the present vacuum-dominated stage is not affected and the early universe dynamics is not significantly altered beyond the standard cosmological model.

VIII. CONCLUSIONS

The cosmological model that we investigate here is based on nonperturbative (in R) vacuum effects of a quantized noninteracting massive scalar field. This model is the sim-

[4]The exit to matter-dominance is analogous to the time-reversed evolution of the present vacuum-dominated stage through the transition at time t_j. Once there is sufficient non-relativistic matter, the exit must occur.

plest one exhibiting these nonperturbative quantum effects. It is based on the extension of quantum field theory to curved spacetime, a subject that has been extensively developed and has given rise to fundamental advances in black hole physics and cosmology. Since our scalar field has no nongravitational interactions, there are no higher order corrections in $\hbar$ to the effective action beyond the order $\hbar$ correction that we work with.

In the present paper, we have studied the particle production resulting from the imaginary part of the same term that, in earlier work, we showed would cause, through its real part, an acceleration of the universe consistent with that inferred from recent supernovae observations and other cosmological data. We find that the predicted rate of particle production during the recent vacuum-dominated era is the same as that found by other methods. It has been shown [24] that this production rate also agrees with that found by the Bogolubov transformation technique [8,31]. This agreement of the particle production rate given by the imaginary part of our effective action, adds confidence to the validity of the acceleration resulting from the real part of the same term in the effective action.

The density and effective temperature of these ultralight particles created after the transition to vacuum dominance is negligible compared to the matter and radiation already present. Long after non-relativistic matter and radiation are sufficiently diluted by the expansion of the universe, the vacuum energy density continues to dominate over the energy density of the created ultralight particles.

We also note that the production of these particles in the early universe would leave a remnant of such particles similar to the remnant of gravitons created near the big bang. The mass of these particles is such that they may contribute a component to dark matter that is sufficiently cold to be gravitationally concentrated in the vicinity of galaxies. The possibility that these may contribute to extended galactic halos is worth exploring.

As is well known, if this scalar field interacted with baryons in a manner similar to an additional neutrino species, this would affect the light element abundances predicted by big-bang nucleosynthesis (BBN), which would place limits on the possible strengths of such interactions. In the present non-interacting model, any of these scalar particles produced during the big bang would not significantly affect the predictions of BBN, just as the presence of primordial gravitons does not alter the predictions of BBN.

The evolution of the scale factor, $a(t)$, in our cosmological solution depends on a single parameter related to the mass m of the particle. Cosmological data give a value for m of order 10^{-33} eV. This mass is of the same order of magnitude as the mass of a pseudo-Nambu-Goldstone boson, obtained by independent particle physics considerations [15]. As discussed in Sec. IV, this fact may be significant with regard to the question of fine-tuning.

We calculated the effective equation of state during the vacuum-dominated era in our model, and gave the function $w(z)\equiv p/\rho$. The effective equation of state of vacuum plus matter during the vacuum-dominated stage turns out to be the same as that of a classical model with radiation plus cosmological constant, although the actual matter content is mainly non-relativistic.

We showed that these nonperturbative vacuum effects of an ultralow mass field would not significantly alter standard cosmology at early times. The possible existence of much more massive scalar fields, for which these nonperturbative vacuum effects would be significant in the very early universe, may enhance inflation. In addition, the associated particle production may help reheat the universe during the exit from early inflation. It is possible that processes, such as spontaneous symmetry breaking (caused, for example, by a self-interaction), may cause the vacuum expectation value of the field to become non-zero, while its fluctuations are reduced. In the present non-interacting model, such processes are absent. The effect of interactions, such as those arising from a PNGB, on the observational predictions of the present model (involving an ultralight mass) is also worth exploring.

Also worth noting is that quantum vacuum terms would become dominant in regions of low average density (i.e., $\rho_m < 2\overline{m}^2\kappa_o$) earlier than in regions of high average density. This would alter the evolution of density inhomogeneities that existed during the early matter-domnated stage of the expansion. This may eventually provide another observational test of our model.

Our present model may also be observationally tested relative to other models through more accurate observations of the small-scale CMBR fluctuations and the SNe-Ia. These fluctuations are determined by the function $w(z)$ given in Sec. VI.[5] Non-zero spatial curvature would, of course, affect the predicted shape of the CMBR fluctuation spectrum, as would the addition of interactions to our present free-field model.

Note added in proof. There is one statement that we made in Ref. [17] concerning the CMBR fluctuation spectrum that we wish to correct. There we stated at the end of Sec. IV that the first acoustic peak in the spectrum would occur in our model at $l \simeq 300$. This statement was inaccurate because we were not careful enough in relating the apparent angular size θ of a fluctuation on the surface of last scattering to the mode number l of the corresponding feature in the CMBR fluctuation spectrum. A comparison of our model with a classical model having a cosmological constant and the same value of Ω_0 as in ours, shows that the value of θ agrees to within 3 percent in both models. Since the latter model with $\Omega_0 \simeq 0.3$ agrees with the observations that the first acoustic peak lies at $l = 197 \pm 6$, our model with the same Ω_0 agrees as well. The corresponding value of m_h is such that our model continues to agree with the other observations that we discussed earlier.

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